Evaluation of Optimization Techniques for Applications in Engineering Design

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Theme

OPTIMIZATION techniques are playing an expanding role in a wide variety of engineering design problems including many aerospace applications. However, a continuing difficulty faced by the engineer interested in solving an optimum design problem is the selection of a suitable optimization technique. Although a few references are now available to guide the designer, much more information on constrained optimization methods is necessary to aid him in the selection process. The purpose of this Synoptic is to summarize the authors' experience with four of the better known techniques for constrained optimization as obtained during the development of a general package of optimization computer programs for use in engineering design. The full paper is an outgrowth of an M.S. thesis by the first author. It contains the details of the study plus listings and a user's manual for the complete package of computer programs. These programs include various constrained and unconstrained optimization routines designed for interchangeable use.

Contents

A convenient summary of previous comparisons of constrained optimization methods has been presented by Himmelblau[1] who discusses work by Colville and Stocker and also presents some additional results. A difficulty with these efforts is that most of the more successful algorithms employed the analytical evaluation of partial derivatives. This is not generally possible in engineering design. Many actual objective functions and constraints cannot be written in straightforward equation form and finite-difference derivative calculations are a common necessity. A recent comparative study by Eason and Fenton[2] has required gradient based search techniques to use finite-difference derivatives. Eason and Fenton considered seventeen different optimization strategies, most employing a type of exterior penalty approach to enforce constraints which did not use a sequential increase in the penalty. The results obtained by Eason and Fenton differ from the results of Himmelblau and from most previous studies of unconstrained optimization in that direct search methods were found to be more attractive than gradient based techniques. Eason and Fenton attribute this to the finite-difference approach required for the gradient computations. This present paper presents contrasting results for the performance of constrained optimization methods and the influence of finite-difference derivatives.

Optimization methods: Minimization problems considered in this effort are inequality constrained and in the form

$$\text{minimize} \quad F(\mathbf{x}) \quad \text{subject to} \quad g_j(\mathbf{x}) \leqq 0, j = 1, 2, \dots, m \tag{1}$$

where $\mathbf{x}$ is the design vector of dimension n, F is the objective function, and the g_j are the m inequality constraints. The approaches to constrained optimization studied here are the methods of feasible directions (FEAS), sequential interior and exterior penalty functions (IPENAL, EXPEN), and a nonsequential exterior penalty approach with direct search (NONSEQ) as found successful by Eason and Fenton. FEAS is based on Zoutendijk's procedure and begins at a point within the constraints using an unconstrained optimization procedure until a constraint is encountered. Then a usable, feasible search direction S is defined, which reduces F and does not violate the constraints, by solving the following problem in linear programing form:

$$\max_{\mathrm{S},\beta} \; \beta \quad \text{subject to} \quad \begin{aligned} &\mathrm{S}^T\nabla F + \beta \leqq 0 \\ &\mathrm{S}^T\nabla g_j + \theta_j \beta \leqq 0 \qquad j \in J \\ &|s_i| \leq 1 \qquad i = 1, 2, \dots, n \end{aligned} \tag{2}$$

where β is a slack variable, T indicates transpose, J is the set of active constraints, and the s_i are the components of S. The θ_j are the "push-off" factors and, as suggested by Fox,[3] are set to one for nonlinear constraints and zero for linear constraints. IPENAL follows essentially the Fiacco-McCormick approach using the modified objective defined as

$$\phi_I(\mathbf{x}, r) = F(\mathbf{x}) - r \sum_{j=1}^{m} \frac{1}{g_j(\mathbf{x})} \tag{3}$$

where r is the penalty parameter. From a starting point within the feasible region the method converges to the constrained optimum of F after a sequence of unconstrained minimizations of ϕ_I with decreasing r. EXPEN is based on the modified objective

$$\phi_E(\mathbf{x}, r) = F(x) + r \sum_{j=1}^{m} \langle g_j(\mathbf{x}) \rangle^2 \tag{4}$$

where

$$\langle g_j(\mathbf{x}) \rangle = \begin{matrix} 0 & \text{for} \quad g_j(\mathbf{x}) < 0 \\ g_j(\mathbf{x}) & \text{for} \quad g_j(\mathbf{x}) > 0 \end{matrix}$$

and r is the penalty parameter. This method converges to the constrained optimum from outside the feasible region for a sequence of unconstrained minimizations with increasing r. NONSEQ uses the exterior penalty function employed by Eason and Fenton and is also in the form of Eq. (4) but the penalty parameter is defined as

$$r = \begin{matrix} 4 \times 10^5 |F(\mathbf{x})| & \text{for} \quad |F(\mathbf{x})| \geqq 1 \\ 4 \times 10^5 & \text{for} \quad |F(\mathbf{x})| < 1 \end{matrix} \tag{5}$$

and is not changed during the optimization process.

The unconstrained optimization method used in FEAS, IPENAL, and EXPEN for this study is the Davidon-Fletcher-Powell technique. All gradient evaluations were carried out by the simple finite-difference computation

$$\frac{\partial f}{\partial x_i} = \frac{F(x_1, \dots, x_i + \Delta, \dots, x_n) - F(x_1, \dots, x_i, \dots, x_n)}{\Delta} \tag{6}$$

generally with $\Delta = 0.001\,|x_i|$ unless $x_i = 0$ where $\Delta = 0.001$ was used. The unconstrained minimization procedure used in NONSEQ is the program PATSH[4] commended by Eason and Fenton. This program employs the Hooke and Jeeves method which is a strong ridge tracking technique and the philosophy here is apparently that a large penalty parameter can be immediately applied with PATSH following the resulting steep walled valley to the constrained optimum, eliminating the need for sequential increases in r and repeated unconstrained minimizations.

Problems: The five constrained optimization problems selected for this comparative study have been taken from Appendix A of Ref. 1 so that interested readers may easily locate the problem

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Table 1 Summary of the problems

Prob. No.	Prob. No. Appendix A[1]	Objective Function	Constraints	F (starting x) F* (optimum x)
1	24	$(x_1-2)^2+(x_2-1)^2$	$x_1^2-x_2 \leq 0$ $x_1+x_2-2 \leq 0$	$F(0.5, 0.5) = 2.5$ $F^*(1, 1) = 1$
2	7	$-0.063y_2(\mathbf{x})y_5(\mathbf{x})+5.04x_1+3.36y_3(\mathbf{x})+0.035x_2+10x_3$	14 nonlinear 6 linear	$F(1745, 12000, 110) = -868.6$ $F^*(1728.37, 16000, 98.13) = -1162.0$
3	10	$\sum_{j=1}^{5} e_j x_j + \sum_{i=1}^{5}\sum_{j=1}^{5} c_{ij}x_i x_j + \sum_{j=1}^{5} d_j x_j^3$	15 linear	$F(0, 0, 0, 0, 1) = 20$ $F^*(0.3000, 0.3335, 0.4000, 0.4285, 0.2240) = -32.349$
4	11	$5.3578x_3^2+0.8357x_1x_5+37.2932x_1-40792.141$	6 nonlinear 10 linear	$F(78.62, 33.44, 31.07, 44.18, 35.22) = -30367$ $F^*(78.0, 33.0, 30.0, 45.0, 36.8) = -30665.5$
5	16	$-0.5(x_1x_4-x_2x_3+x_3x_9-x_5x_9+x_5x_8-x_6x_7)$	13 nonlinear 1 linear	$F(0.5773, 0.6506, 0.3678, 0.4881, -0.0370, -0.0692, 0.6347, 0.8989, 0.1404) = -0.0532$ $F^*(0.9971, -0.0758, 0.5530, 0.8331, 0.9981, -0.0623, 0.5642, 0.8256, 0.0) = -0.8660$

details. Two of the problems (3 and 4) were also considered by Eason and Fenton. A summary of all the problems is shown in Table 1; feasible starting points were used for each optimization.

Results: The optimization program package has been arranged so that an optimization problem is solved by making repeated calls to a user-prepared subroutine. This subroutine computes the values of the objective function and all constraints for any set of design variables specified in the optimization procedure. The modified objectives needed for the penalty function methods are included in the optimization package and finite-difference gradient computations can be made automatically. User preparation for all of the optimization methods considered is the same and is primarily the writing of the subroutine for computing the objective function and constraints from the design vector. Additionally, a short mainline program must be written which calls the desired constrained optimization method specifying the initial values of the design vector, values for any adjustable parameters, etc.

Each of the methods contains a parameter or two which must be adjusted to obtain good performance on a particular optimization problem. The results of this study in Table 2 are those obtained after one or two "tuning" runs and are believed to be the results of most interest to an engineering designer. Stopping rules used by the various techniques have been adjusted so that termination occurs as soon as a realistic approximation to optimum has been achieved. All intermediate output information was omitted in the final computer runs so that the results of Table 2 include only essential computational effort. To allow the comparison of computation times on different computers, a standard timing program has been devised by Colville and its execution time may be used to normalize the time required for a given problem. This standard program has been reported to execute in 20.5 sec on the CDC 6400.[1]

Conclusions: The conclusions which can be made from a study of the results in Table 2 are in agreement with the conclusions drawn by the authors on a less quantitative basis after seeing these optimization methods applied to a variety of design problems. These conclusions are as follows.

1) The method of feasible directions can solve smaller problems containing mostly linear constraints quite rapidly and effectively. The method, however, may have significant difficulties when larger numbers of design variables or nonlinear constraints are involved.

2) The sequential interior and exterior penalty function methods work well with a gradient based search procedure using finite difference gradient computations. EXPEN shows slightly better performance than IPENAL and also has the advantage that it may be started without finding an initially feasible design.

3) The use of a nonsequential exterior penalty function with the Hooke and Jeeves pattern search is well suited for smaller problems, particularly with complicated functions or constraints as in problem 2 that may be discontinuous or have discontinuous gradients. However, the method does not converge as well with larger numbers of variables as the sequential penalty methods considered here.

On the basis of computer times normalized by Colville's standard timing program, the results obtained here for NONSEQ on problems 3 and 4 agree reasonably with Eason and Fenton's results for the same problems. However, gradient based searches with finite-difference computations were used successfully in this study and the difficulties reported by Eason and Fenton were not experienced. It is believed that the careful use of sequential penalty functions accounts for this difference since finite-difference gradient evaluations may compound the valley tracking problem which can make any optimization difficult. The use of penalty functions, of course, creates such valleys and when the penalty parameter is large for exterior penalties or small for interior penalties the walls of these valleys are very steep. A typical optimization strategy will quickly move to the constraint in such a case but will have difficulty moving along it to the constrained optimum. A conservative initial choice of the penalty parameter allows the first unconstrained optimization to converge on a moderate valley displaced from the constraints. Then proper sequential adjustment of the penalty parameter allows the constrained optimum to be approached by shifting the valley closer and closer to the active constraints as its walls become steeper and steeper. Each new unconstrained optimization moves primarily down the side of the valley and motion along the floor is minimized. The increased convergence speed of the Davidon-Fletcher-Powell method makes these precautions and the series of unconstrained optimizations worthwhile for problems of considerable size.

Table 2 Execution times (CDC 6400)

Problem No.	FEAS	IPENAL	EXPEN	NONSEQ
1	1.14 sec	1.53 sec	1.39 sec	1.31 sec
2	S[a]	23.6	15.6	4.85
3	2.14	4.11	3.32	49.3
4	2.92	3.02	2.30	29.3
5	F[b]	4.35	2.87	F[b]

[a] S = slow. [b] F = failed.

References

[1] Himmelblau, D. M., *Applied Nonlinear Programming,* McGraw-Hill, New York, 1972, pp. 366–392.

[2] Eason, E. D. and Fenton, R. G., "A Comparison of Numerical Optimization Methods for Engineering Design," *Transactions of the ASME: Journal of Engineering for Industry,* Vol. 96, No. 1, 1974, pp. 196–200.

[3] Fox, R. L., *Optimization Methods for Engineering Design,* Addison-Wesley, Reading, Mass., 1971, p. 227.

[4] Whitney, D. E., private communication, MIT, Cambridge, Mass.